MATH 3100: INTRODUCTION TO PROBABILITY

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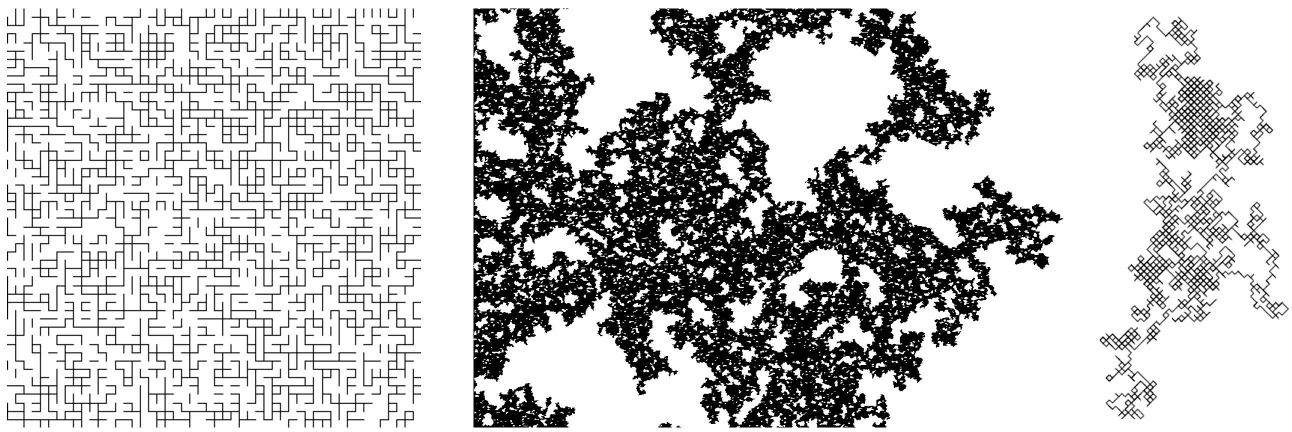
1. *A mathematical study of randomness*

How random is everything around us, and what chance do we have of understanding it? What to do when you're not certain, and how to do it right? How many falling stars will you see as you walk outside one beautiful night?

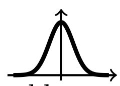
Probability theory is a mathematical study of uncertainty. It is a rigorous foundation of statistics — and many areas of human knowledge operate in a language of statistics nowadays (yes, and robots use it, too!). The course introduces fundamental concepts, ideas, and techniques of probability theory. It will provide you with foundational mathematical knowledge needed to address the questions above, and will help you develop intuition about randomness.



Examples of random structures: bond percolation `close-up` (left), at a `larger scale` (center), and a `random walk` (see also a `simulation` of a random walk). Note: this PDF has green clickable links, like in the previous sentence.

What you will get from this course.

1. Mastery of basic probability concepts:
   (a) What is a probability space and how to translate commonly-sounding problems into this language;
   (b) How to count (in an advanced way) to compute probabilities;
   (c) What is a random variable, a probability distribution, and what are their main quantitative properties;
   (d) How commonly encountered probability distributions (binomial, Poisson, exponential, Gaussian) look like and behave, what are their properties, and in which situations they typically arise.

2. How large random systems behave, and what the bell-shaped curve  has to do with this.

3. How to describe and quantify mutual dependence of random events, and how to use such a description to infer properties of "hidden" random events.

Date: Tuesday 28th March, 2017, 19:32.

An up to date syllabus is always on `GitHub` at `https://github.com/lenis2000/Syllabi/blob/master/Syllabus_3100_s17.pdf`. For direct PDF download use `this link`. LaTeX source with *changes* to the syllabus is `here` (click "History").

4. How to apply probability theory to model real-life processes like queues (consisting of people or requests at an internet server).
5. How to collaborate on solving probability problems in pairs, small groups, and online, and present solutions clearly and efficiently.
6. How to design probability problems (for example, for the final exam), and evaluate problems presented by others.
7. In what ways probability theory is connected to science, engineering, and other branches of knowledge.

Prerequisite. You should have taken at least one semester of calculus, because the study of random variables often requires single and double integrals and infinite series.

What this course is and what it is not. This course in probability *theory* belongs to pure mathematics, with rigorous definitions, calculations, and proofs. However, the objects which we study are motivated by real-life applications, and so pure mathematical arguments often appeal to our common sense understanding of these objects. There will be opportunities to explore (and discover new) connections of the theory studied in the course with real world.

Also, this course does not thoroughly discuss *applications to statistics* (except maybe in the group project, see Section 3.3). Probability theory focuses on developing the mathematical side, and statistics applies these mathematical theories to real data (coming from observations). In this course we will not discuss how to analyze data coming from observations — there are courses in statistics for that.

2. *Necessary information*

2.1. Meeting times.

Class times	MoWe, 2:00PM – 3:15PM Monroe Hall 124
Midterm	We, February 22, 2:00PM – 3:15PM Monroe Hall 124
Final exam	Mo, May 8, 2:00PM – 5:00PM Monroe Hall 124

Instructor: Leonid Petrov
Email: We use `Slack` instead, see Section 4
Office: 209 Kerchof Hall
Office hours: We 12:50-1:50 and 3:30-4:30 (on most of the weeks), and Thu 2:00-3:00 (more reliably than We); or by appointment (you can make as many appointments as you want; I am also always available on `Slack`)

2.2. **About the instructor.** I am an assistant professor in the Department of Mathematics at UVA, and I've been here since 2014. My research area is probability theory (very appropriate for this course!). More precisely, I am using exact formulas to study large random systems. I also like computer simulations of random systems, some examples are `at this link` or at my office door. I'll be happy to tell you more if you're interested.

I also co-organize the UVA Math Club (`http://pi.math.virginia.edu/mathclub/`), let me know if you'd like to be added to the Math Club mailing list.

2.3. **Textbook.** "Probability" by Jim Pitman. See also Section 5 below for discussion of how we'll use the textbook, and for other helpful resources.

3. *Assessing your learning*

Learning mathematics means *doing* mathematics: during class meetings, on your own, and in groups. In this course, doing mathematics mainly amounts to solving problems. There are five aspects which are assessed in this course:

3.1. Homework (20%).

- Weekly homework will consist of textbook and other problems aligned with lectures, to help you practice new concepts and techniques. The homeworks are usually due on Mondays, and will be assigned at least a week before the due date.
- You are encouraged to work together on homework assignments (also can do it online via `Slack`, which allows private groups of up to 9 people), to take advantage of the challenge-defend discussions that help understand things better. However, each student needs to submit her/his own homework assignments, and should work individually when writing them up to demonstrate the understanding of the material.
- The homeworks are graded "coarsely", that is, each homework will be assigned one of four grades:

Grade	VG (very good)	G (good)	OK	N
	All problems solved correctly with minor issues like arithmetic mistakes, and solutions explained in full detail	Most problems solved correctly, and solutions explained in reasonable (close to full) detail	More than 3/4 of problems attempted, many solutions are incorrect, incomplete, or not explained in detail, but the work displays adequate understanding of most of the material	Work not submitted on time, or less than 3/4 of problems attempted, or most solutions are incomplete, or work clearly displays lack of understanding of most of the material
%	100%	90%	75%	0%

  It is expected that most students who put reasonable effort into the homework will get VG or G grades.
- The homeworks *must be submitted only on Collab* (i.e., hard copies are not accepted). Take pictures or scan your work, make sure it's readable, put it into a *single PDF file with correct orientation*, and upload it before the deadline. Submitting work like this has many benefits: (1) you retain a paper copy to prepare for tests; (2) your submitted work is never misplaced or lost, and there is a digital trail; (3) the grading will be much faster and will allow me to immediately incorporate my impressions of the homeworks into the class discussions. Moreover, (4) knowing how to scan and make a single PDF file is a valuable technical skill for the future: ask me and I can help you learn it.

3.2. Course engagement / quizzes (12.5±5%). This includes short pop quizzes during lectures (at random days), and activity in `Slack` (including answering other student's questions).

3.3. Group project (12.5±5%). [1]There will be a group project on applications of probability theory. Its purpose is to put you in a collaborative environment, and help you explore how probability theory can be applied to real-world situations. In fact, nowadays applications of probability theory (along with statistics, and sometimes under the names of modeling or operations research) span almost every area of human knowledge and activity, so there is a lot to choose from.

There will be about 10 groups. There are two tracks of this project for each group to choose from: you can either make a *computer simulation* of an interesting random system and then experimentally describe its behavior; or to come up with a probabilistic model of a real-world phenomenon, and use the probabilistic *analysis* of the model to quantitatively understand it. In the end each group will produce a report which will be peer-reviewed by other groups, and will give a short presentation on the project.

3.4. Midterm test (20%). The midterm test during regular class time on February, 22 will have similar taste as homeworks, and will test basic knowledge of the material.

A two-sided letter size formula sheet, hand-written by yourself, will be allowed on the midterm test and the final exam. Preparing this formula sheet will help you review the material, and paint a systematic picture in your head. Formula sheets cannot contain any photocopied or printed material

[1]The course engagement and group project together add up to exactly 25%, but I am not yet sure how these 25% will be distributed; in particular, this depends on the number of pop quizzes we end up having.

— do everything by hand (of course, you can include any theorems, formulas, pictures, examples, etc). The use of calculators (but not in mobile phones) is allowed on midterm test and the final exam.

I encourage you to collaborate on test preparation, but needless to say that during the test and the exam each student must work individually.

3.5. **Final exam (35%).** The final exam will be cumulative, but will put more focus on topics covered after the midterm.

Letter grades. The scale by which course percent grades are turned into course letter grades will most likely be the following:

Grade	$A+$	A	$A-$	$B+$	B	$B-$	$C+$	C	$C-$	$D+$	D	$D-$
Minimum %	100	93	89	86	82	79	76	72	69	66	62	59

However, I reserve the right to slightly change this grade scale after the final exam to better incorporate into the letter grade the possible fluctuations in the difficulty level of problems in the midterm and the final exam.

4. *Communication*

4.1. **`Slack`.** My email is petrov@virginia.edu, but for the communication we will use `Slack` — an industrial standard of work messengers, with a web version and apps for all platforms. This will make me more accessible if you have questions, and also will let you answer questions of your fellow students. There will be course content available *only* on `Slack` — my lecture notes which I prepare for each class.

The course "team" is at `https://3100-s17-uva-petrov.slack.com`. You'll get an invitation to join by e-mail, and it is expected that you register. Please let me know if you have issues with access. It is also expected that you will check announcements, and will participate in (or at least read) the discussions of the course material.

Some things to note:

- **Direct messages**: There are direct messages where you can ask me questions one-on-one. You can also create private groups with up to 9 people, which is good for homework collaboration (but read Section 5.5 on collaboration).
  Private channels (containing myself) invisible to other students will be created for each group for discussions of the group project.
- **Privacy**: Although `Slack` is a messaging app, it should be used professionally, especially in public discussions. The app supports private direct and group messages. But please note that in principle the admin (i.e., myself) can obtain access to **all** direct messages between members of the team. The procedure would involve sending a paper request via the usual mail, and everyone will be notified if the direct messages are accessed — so this can happen only in extreme circumstances.
- **Email address**: You need an email address to use `Slack`, and it may be visible to other students. Normally it's your UVA email address (I'll send an invite there), but if you are not comfortable sharing your UVA email, let me know and I'll be happy to send invite to another address. And you can later change the email address yourself in the settings.
- **Notifications**: With `Slack`, it is very easy to not miss important announcements and direct messages, even if you don't check it all the time — under the default settings, you will get notified (by email, too!) about direct messages and mentions of your name. All announcements will have mentions of everybody to draw attention. Also, you can change notification settings, but please remember that there will be sometimes very important activity in `Slack` that you don't want to miss.
- **Public discussions**: Public messaging is separated into channels (#general for class-wide questions/answers where you are welcome to post; #announcements and #class-notes where I will post relevant information; special channels for midterm and final preparation, etc.).
- **Many other features**: `Slack` supports reminders out of the box. You can ask it to remind yourself about a particular message, or just set a general reminder. With this, it is easy to keep track of important messages (like class announcements or problems I'll post there). There are numerous third-party plugins one can add to `Slack`, and if you can think of good ones to add, please let me know so I can add them.

4.2. **Collab.** Solutions to homeworks / quizzes / midterms will be posted to both `Slack` and Collab. Grades will be posted to Collab as usual. Homework assignments will be posted to Collab and will be also announced in `Slack`'s #announcements channel. Your homework solutions can be submitted only through Collab, and will be graded there.

If you have anonymous comments on anything related to the course, you can make them via Collab.

5. *How to succeed in the course*

5.1. **General things.** The best way to learn in the course is to come to all lectures, retain good memories of what was in there (brief class notes which I will prepare for each lecture will be made available on `Slack`), and do all the homework problems on your own or in collaboration. This will prepare you well for pop quizzes and tests, and will help you in doing the group project.

5.2. **Textbook and other resources.** The textbook *"Probability" by Jim Pitman* is an excellent resource to gain understanding of the course material. Some notes about it:

- You don't have to bring the textbook to all classes, although this may be helpful sometimes.
- I strongly encourage you to read the textbook. It includes many examples and extra exercises which augment the concepts discussed in class.
- The textbook contains much more material than will be covered in classes, so it makes sense to come to all classes and see which parts of the textbook are covered and which are omitted.

There is another (free) *textbook* with more problems and more material. It may be helpful if you want a deeper understanding of some concepts, or if you want to read exposition of the familial material in a different style (which might be very helpful for better learning!):

> Download: https://math.dartmouth.edu/~prob/prob/prob.pdf
> Accompanying web page: https://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/book.html

There is a number of online resources which may help you while doing the homework: Khan Academy, Wikipedia, and many other places contain lots of basic material on probability theory. Google is also a valuable resource.

5.3. **Office hours and `Slack`.** I am available during office hours to answer questions on the content of the course, clarify various points, and I can also help you with the homework assignments.

Moreover, I am generally available in `Slack` for quick questions. Note that if you post a question to the #general channel, you also have a chance to get help from your fellow students!

5.4. **Tutoring center.** The Math Department Tutoring Center is available for helping students in this course: see http://people.virginia.edu/~psb7p/MTCsch.html for more information and schedule.

5.5. **Collaboration on homework assignments.** Group work on homework problems is allowed and strongly encouraged (unless otherwise stated for a particular assignment). Discussions are in general very helpful and inspiring. Nevertheless, before talking to others, get well started on the problems, and contribute your fair share to the process.

When completing the written homework assignments, everyone must write up his or her own solutions in their own words, cite any reference other than the textbook and class notes. Quotations and citations are part of the Honor Code for both UVa and the whole academic community.

It is very important that you truly understand the homework solutions you hand in, otherwise you may be unpleasantly surprised by your in-class test results.

If you work together on homework, please write the names of your collaborators on the front.

5.6. **Collaboration on the group project.** Collaboration is required to complete the group project, and the write-up of the report can also be done in group. However, it is important that every member of the group understands the nature and results of the project.

6. *Approximate course schedule*

Add/drop information: http://www.virginia.edu/registrar/reginst1158.html#Deadlines

NOTE: Please don't make travel plans conflicting with midterm (2/22) or final exam (5/8)

The course has 3 "pillars": central limit theorem for Gaussian approximation, Poisson processes, and conditional expectations. Plus there are several technical things to learn: random variables, expectations as integrals, joint distributions, etc.

[**week 1**] 1/18. Introduction. What is probability theory. Sections 1.1–1.3

[**week 2**] 1/23, 1/25. Advanced counting. Conditional probability and independence. Section 1.4

[**week 3**] 1/30, 2/1. Advanced counting. Bayes' rule. Repeated trials. Sections 1.5, 1.6, and 2.1.
2/1: discussion of the group project: real-world applications of probability theory

[**week 4**] 2/6, 2/8. Binomial distribution. Gaussian approximation. Sections 2.1, 2.2
By 2/8, form groups up to 6 students for the project. Group lists due. Slack groups will be created

[**week 5**] 2/13, 2/15. Random variables. Expectation and conditional expectation. Sections 3.1, 3.2, 3.3, and 6.1
By 2/15, choose a real-world problem to analyze. Report this to the class in a 2-minute presentation.

[**week 6**] 2/20, **test on 2/22**. Poisson distribution and Poisson process. Sections 2.4, 3.4, and 3.5
Sections covered in the test: 1.1–1.6, 2.1, 2.2, 2.4, 3.1–3.3, 3.5

[**week 7**] 2/27, 3/1. Poisson distribution and Poisson process. Coin tossing and geometric distribution. Sections 2.4, 3.4, and 3.5
By 3/1, choose track: analysis or simulation, and formulate a draft problem/model associated with your real-world setting. Short 1-page description of the problem due.

Spring break

[**week 8**] 3/13, 3/15. Variance, Chebyshev inequality, central limit theorem (normal approximation). Continuous distributions. Sections 3.3, 4.1.

[**week 9**] 3/20, 3/22. Continuous distributions. General Gaussian approximation. Poisson processes, geometric, exponential and gamma distributions. Sections 2.4, 3.3, 3.4, 3.5, 4.1, and 4.2.
By 3/25, formulate a draft solution/simulation of your problem in the simplest interesting case. Draft of the group report due. Then continue working on how to extend the problem to other cases, to include more realistic situations.

[**week 10**] 3/27, 3/29. Continuous distributions. Change of variable and cumulative distribution function. Sections 4.4 and 4.5

[**week 11**] 4/3, 4/5. Joint and conditional distributions. Introduction. Sections 5.1 and 5.2
4/8: Final version of the group report due

[**week 12**] 4/10, 4/12. Joint and conditional distributions. Sections 6.1–6.3
4/17: Peer reviews due

[**week 13**] 4/17, 4/19. Joint and conditional distributions. Sections 6.1–6.4.

[**week 14**] 4/24, 4/26. Joint and conditional distributions. Bivariate normal distribution. Application to statistics. Correlation. Sections 5.3 and 6.5

[**week 15**] 5/1.
1-2 of the last classes — group presentations

7. *Policies*

7.1. **Laptops and smartphones.** Please do not use laptops and smartphones during the class. You won't need them to participate in the discussions, but they may easily distract you or other students (or me!). If you absolutely must use a laptop, please sit in the back row.

7.2. `Slack` **conduct.** Although `Slack` is a chatting app, it should be used professionally, especially in public discussions. The app also supports private direct messages and I encourage to use them to collaborate on homework problems and projects. But please note that in extreme circumstances the admin (i.e., myself) can obtain access to **all** direct messages between members of the team. So just don't say anything on `Slack` that you wouldn't say in class.

7.3. **Late/make up work.** Each assignment will have due date and time. Late assignments are not accepted. There will also be no make up for the midterm test. However, if you have special needs, emergency, or unavoidable conflicts, please let me know as soon as possible, so we can arrange a workaround.

7.4. **Honor Code.** The University of Virginia Honor Code applies to this class and is taken seriously (in particular, see Section 5.5 on homework collaboration). Any honor code violations will be referred to the Honor Committee.

7.5. **Special needs.** All students with special needs requiring accommodations should present the appropriate paperwork from the Student Disability Access Center (SDAC). It is the student's responsibility to present this paperwork in a timely fashion and follow up with the instructor about the accommodations being offered. Accommodations for test-taking (e.g., extended time) should be arranged at least 5 business days before an exam.