MATH 3340: COMPLEX VARIABLES WITH APPLICATIONS

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1. Complex variables

Complex analysis is a central part of Mathematics. Many concepts work easier and much more natural in the complex setup:

- For example, if a function f(z) of the complex variable z has one derivative at a point z_0 , then it has infinitely many derivatives, and possesses a power series (Taylor) expansion at z_0 , which converges to our function. Compare this with the "bad" behavior of the function $f(x) = e^{-1/x}$ for x > 0 (and f(x) = 0 for $x \le 0$) of the real variable x, which has infinitely many derivatives, but whose Taylor series at 0 is identically zero.
- Any algebraic equation, even $x^8 + 1 = 0$, has a solution over the complex numbers (even if no real solutions). In fact, the equation $x^8 + 1 = 0$ has 8 different solutions, and they all can be illustrated by vertices of a perfect octagon in the complex plane.

The course is centered around the basics of the theory of functions of a single complex variable.

After taking this course, you will be able to solve problems and understand the basics of complex numbers, analytic functions, complex integration, Cauchy formulas, power series, residues, and conformal mappings. Moreover, you will learn how to apply these tools to other parts of Mathematics, and to some physical models.



Real part of a particularly complex function (left), and its contour plot (right).

Prerequisites. Good command of single and multivariable calculus at the level of MATH 1310, 1320, and 2310.

Date: Compiled on Saturday 18th January, 2020, 07:39 (in whatever timezone I was at that time).

An up to date syllabus is always on GitHub at https://github.com/lenis2000/Syllabi/blob/master/Syllabus_ 3340_s20.pdf. For direct PDF download use this link. LATEX source with *changes* to the syllabus is here (click "History").

Note that this PDF has green clickable links.

2. Necessary information

Class times: TuTh 12:30PM - 1:45PM in New Cabell 309

Exams: Please do not make travel plans which conflict with the midterms or the final exam.

- Midterm 1: In-class on Thursday, February 6 (class time, New Cabell 309).
- Midterm 2: In-class on Tuesday, April 7 (class time, New Cabell 309).
- Final exam: Tuesday, May 5, 2-5 (New Cabell 309).

Instructor: Leonid Petrov

Email: petrov@virginia.edu or lenia.petrov@gmail.com

Office: 209 Kerchof Hall

Office hours: The default times I am in office are Tuesdays and Thursdays, 9:30-10:30, except the weeks when I'm traveling.

You are welcome to make an appointment and meet outside the usual office hours. For this, please use the online tool located at https://lpetrov.cc/teaching/. (I am automatically available during office hours — and you cannot schedule appointments online for those times.) You can make as many appointments as you want.

Course webpage: I will set up a collab page for homework submissions and course materials.

3. Course materials

The textbook is "Fundamentals of Complex Analysis" (3rd edition) by Saff and Snider, Pearson, ISBN-10: 0139078746. We will discuss material from Chapters 1–6, and selected topics from Chapters 7–8.

4. Assessing your learning

Learning mathematics means *doing* mathematics: during class meetings, on your own, and in groups. In this course, doing mathematics mainly amounts to solving problems. Below are the concrete aspects which are assessed in this course:

4.1. **Homework.** Weekly homework will consist of problems aligned with lectures and quizzes, to help you practice and enrich the material presented in class. Putting an adequate effort into solving the homework problems and communicating your solutions clearly is of paramount importance for your learning. The homeworks are due **in class** on the specified date, and will be assigned at least a week before the due date. Please **put your problems in order**, indicating clearly which problems you're skipping — this will greatly help with the grading.

Homework solutions are posted soon after the homework deadline, so late work cannot be accepted. The lowest homework grade will be dropped. The homeworks are graded "coarsely", that is, each homework will be assigned one of four grades:

Grade	VG (very good)	G (good)	OK	N
	All problems solved correctly with minor issues like arithmetic mistakes, and solu- tions explained in full detail	Most problems solved correctly, and solutions explained in reasonable (close to full) detail	More than 3/4 of problems at- tempted, many solutions are incor- rect, incomplete, or not explained in detail, but the work displays adequate understanding of most of the material	Work not submitted on time, or less than 3/4 of problems at- tempted, or most so- lutions are incom- plete, or work clearly displays lack of un- derstanding of most of the material
%	100%	90%	75%	0%

It is expected that most students who put reasonable effort into the homework will get VG or G grades.

Note on collaboration on homework assignments. Group work on homework problems is allowed and encouraged. Discussions are in general very helpful and inspiring when learning mathematics. Nevertheless, before talking to others, get well started on the problems, and contribute your fair share to the process.

When completing the written homework assignments, everyone must write up his or her own solutions in their own words. It is very important that you truly understand the homework solutions you hand in, otherwise you may be unpleasantly surprised by your in-class test results.

Needless to say that when working on in-class assignments (quizzes, tests) you are required to work alone.

4.2. Quizzes. There will be short quizzes (10-15 minutes) during the classes at random days. They will test the previous week's material and/or recent homework topics. Quizzes are not announced in advance, and there can be two quizzes on a given week.

You should view quizzes as testing your "work in progress", which will allow me to adjust the pace of the course. For this reason, the overall quiz grade is included in the same "bucket" with class participation and office hours discussion, see below.

4.3. Midterm tests and the final exam. The midterms and the final exam will feature problems modeled after homework. The final exam is comprehensive, with a focus on the last part after the second midterm.

The exams will be aimed at checking not so much memorization and routine computational skills, but rather understanding of fundamental concepts and principles and the ability to apply the material learned to solving various problems, including those a student might have never seen before. A missed exam gives a score of zero, unless a student has contacted the instructor a week in advance and agreed upon a procedure to make it up. Under the rules of the College, early examinations are not permitted.

4.4. How to succeed in the course. The best way to learn in the course is to come to all lectures, take good notes (some notes may be provided), ask many questions, do all the homework problems, and express your solutions clearly. This will prepare you well for quizzes, midterms, and the final exam.

Mathematical questions are appreciated and encouraged any time during the class. Please use the office hours as much as possible for additional clarifications and occasional homework help. Remember that I am available outside of office hours by appointment which you can book at https://lpetrov.cc/teaching/

4.5. Grade distribution. Your grade will consist of:

- Homework 20%, lowest homework dropped
- Quizzes, class participation, office hours discussion 15%, one or two lowest quizzes dropped
- Midterms — 15% each
- $\bullet\,$ Final exam — 35%

The score above 90% is usually enough for an A. The score below 50% usually means failing. Other factors such as in-class participation and improvement over time may impact positively your final grade. Excessive absence may lower the final grade.

5. Policies

5.1. Laptops and smartphones. Please do not use laptops and smartphones during the class. You won't need them to participate in the discussions, but they may easily distract you or other students (or me!). If you *absolutely* must use a laptop (for typing up the lecture notes), please sit in the back row.

5.2. Late/make up work. Each assignment will have due date and time. Late assignments are not accepted. There will also be no make ups for the midterm tests and the final exam. However, if you have special needs, emergency, or unavoidable conflicts, please let me know as soon as possible, so we can arrange a workaround.

5.3. Honor Code. The University of Virginia Honor Code applies to this class and is taken seriously. Collaboration on homework assignments is allowed within the bounds discussed above in the corresponding section. Any honor code violations will be referred to the Honor Committee.

5.4. **Special needs.** All students with special needs requiring accommodations should present the appropriate paperwork from the Student Disability Access Center (SDAC). It is the student's responsibility to present this paperwork in a timely fashion and follow up with the instructor about the accommodations being offered. Accommodations for test-taking (e.g., extended time) should be arranged at least 5 business days before an exam.